

Coin Toss Problem

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Problem:

Given a coin with probability p of landing on heads after a flip, what is the probability that the number of heads will ever equal the number of tails assuming an infinite number of flips?

Solution:

Think of this problem instead as one of a random walk along a number line. Let 0 be the starting point, p be the probability of moving to the right, and q be the probability of moving to the left.

Let B = probability of ever moving one to the left from where you are.

Let A = probability of revisiting current square from the right.

$$B = q + Aq + A^2q + A^3q + \dots = q/(1-A).$$

$$A = pB \rightarrow B = A/p \rightarrow B = A/(1-q).$$

$$q/(1-A) = A/(1-q) \rightarrow A = q, 1-q.$$

However, A must be less than or equal to both p and q , thus the reasonable solution is $A = \min(q, 1-q)$.

Redo the above only reverse the words left and right and A will still equal $\min(q, 1-q)$. One ramification of this is that the probability of revisiting 0 is the same from both the left as the right. This stands to reason since any path has a mirror image on the other side of equal probability. So the answer is $2 * \min(q, 1-q)$. Where I am a little uncomfortable is dismissing the other solution of A . I believe I can do so but can not put into words why.